Higher order thinking with Venn diagrams

BY DAVID WALBERT

Teachers know that graphic organizers are powerful ways to help students understand complex ideas. Drawings and diagrams engage visual learners; they show relationships, clarify concepts, and facilitate communication.

You're probably familiar with the Venn diagram, or "double bubble chart." Research has shown that identifying similarities and differences is perhaps the single most powerful strategy for student learning.

But comparing two items — whether animals, countries, or literary characters — is only the beginning. By adapting and building on the simple double bubble chart, you can diagram classification systems that encourage students to recognize complex relationships between items and characteristics. This article will show you how.

Getting started: Compare and contrast

We all know that graphic organizers can be a tremendously helpful learning tool, which is why the Venn diagram or "double bubble chart" is so popular in identifying similarities and differences. Let's say we're talking about animals that live in water. Here's a "double bubble chart" we might draw in an elementary classroom:

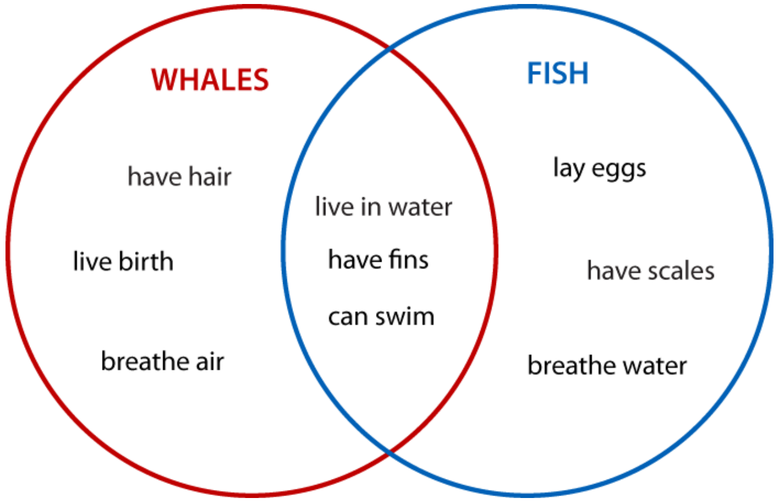

Figure 1. Figure 1. Comparing whales and fish.

In this diagram we have two circles, each representing one thing or kind of thing — in this case whales and fish. Inside the circles we list words that describe those things. If a word describes both things we put it in the middle, in both circles. All Venn diagrams share this basic structure: They are composed of overlapping circles (or other shapes) with words written in them. But, as we'll see, they can get much more complicated, and they can be used for different purposes.

I've used animals here because they're a subject we all understand, but of course you can use this for any subject at any grade level. So, for example, high school English literature: Here's a diagram comparing character traits of two protagonists from Thomas Hardy novels.

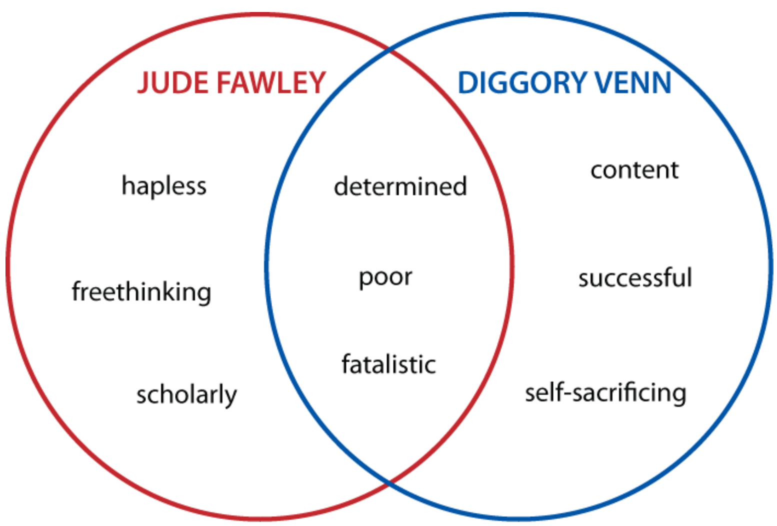

Figure 2. Figure 2. Comparing Thomas Hardy protagonists.

But not everybody enjoys Hardy as much as I do, so we'll just stick with animals for now.

COMPARISON DIAGRAMS

I'm going to call this kind of diagram a *comparison diagram* because, obviously, we're using it to compare two things. Could we use it to compare three things?

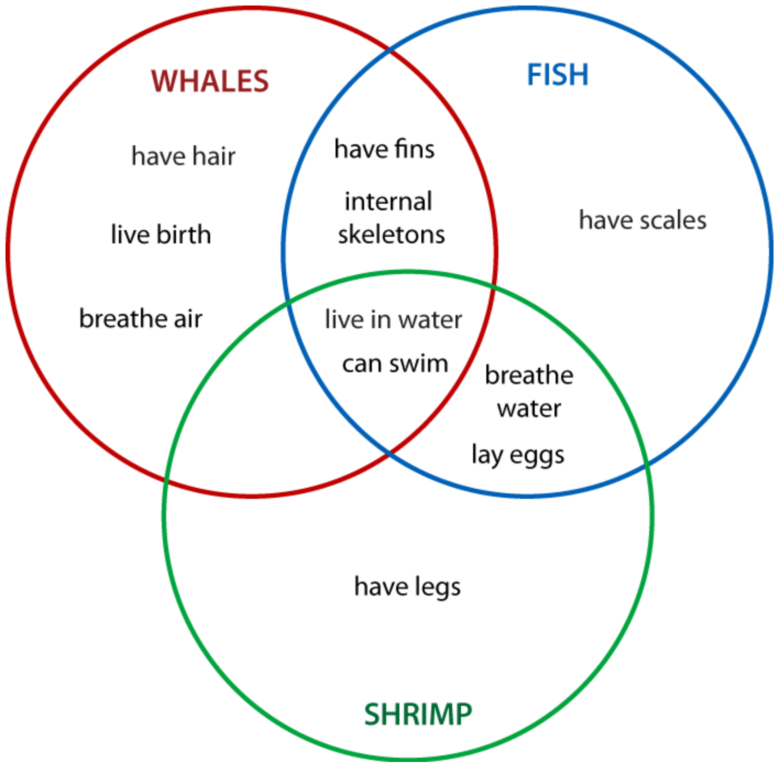

Figure 3. Figure 3. Comparing whales, fish, and shrimp.

Well, sure. I've added shrimp to the animal comparison diagram. As we can see, shrimp are very different from both whales and fish, and they don't seem to have much in common at all with whales. A student might look at this and think, *That's interesting — three very different kinds of animals that live in the water. It looks like quite a lot of different animals live in water.* So our graphic organizer is helping us to understand similarities and differences among animals that have one key trait in common.

But we haven't nearly exhausted the possibilities. What about oysters? Or sea anemones? Or sea turtles? Could you compare four things?

As it turns out, it's mathematically impossible to draw four circles, each of which partially overlaps each of the others. There's a way to do it with shapes other than circles, as we'll see later, but even so, if we want to compare a whole bunch of different animals, drawing individual shapes for each animal is going to be a lot of work, and it will quickly get confusing — it will lose its power as a graphic organizer.

Classification

If we want to go beyond two or three fairly similar things and talk about all the animals that live in water, what we're really doing is *classifying* things. Instead of identifying similarities and differences between two things, we need to establish *categories* of things, and once we've done that, we can place lots of things in them.

CHARTS

Suppose we establish some categories for our water-dwelling animals. Let's take all of the characteristics of these animals out of the diagram and turn them into a chart.

Table 1. Classifying animals that live in water.

have fins	**have legs**	**have internal skeletons**	**breathe air**	**can swim**	**lay eggs**
whales fish	shrimp sea turtles	whales fish sea turtles	whales sea turtles	whales fish shrimp sea turtles	fish shrimp sea turtles

Here, each of the characteristics of water-dwelling animals has been made into a column header, and each of the animals is listed beneath them. (I've added sea turtles because I like sea turtles, and because I don't have to add another circle for them.) Now, I'm *classifying* sea animals.

But this chart has a couple of problems. I've listed each animal multiple times, first of all — that's inconvenient, and hard to read. I can see very quickly which animals share a particular characteristic, such as breathing air, but I'd like to be able to see at a glance all of the characteristics of a particular animal, as well.

Table 2. Classifying animals that live in water.

	have fins	**have legs**	**have internal skeletons**	**breathe air**	**can swim**	**lay eggs**
whales	X		X	X	X	
fish	X		X		X	X
shrimp		X			X	X
sea turtles		X	X	X	X	X
oysters						X
sea anemones						
sharks	X		X		X	X

	have fins	have legs	have internal skeletons	breathe air	can swim	lay eggs
jellyfish					X	X

Now each animal is listed only once, and I've used an X to mark animals that fit a given category. That's easier to read, and it's easier to add more animals (as I've done).

There are still problems with this, though. It's hard to see *connections* among the animals — we've classified them, but we've lost the sense of similarities and differences, haven't we? You can read it across — the characteristics of one animal — or down — the animals that share one characteristics — but it's hard to move around visually within the boxes.

We're also stuck inside this box. When we see these headings running along the top and the left-hand side, and that's all we're likely to think about. I'd like, literally, to think outside the box! What we need is a graphic organizer for classification.

CLASSIFICATION DIAGRAMS

What you need, in fact, is another kind of Venn diagram, which I'll call a *classification diagram*.

Let's go back to our simple animal example.

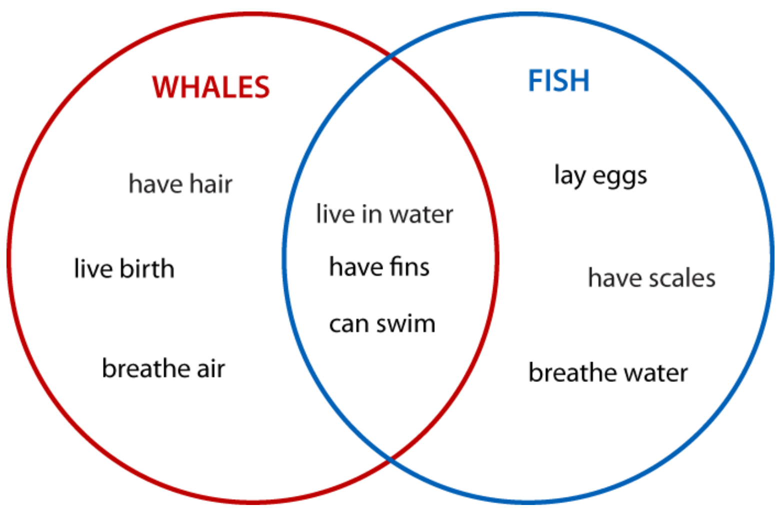

Figure 4. Figure 1. Comparing whales and fish.

The truth is, this is not a Venn diagram. Mathematically speaking, a Venn diagram represents *sets* and *elements*. The circles are sets, and the items in the sets are elements of those sets. The space in the middle, where the circles overlap, is the *intersection* of the sets. You've probably heard these terms before and they're fairly intuitive, but we don't use the formal language very often. And while you don't have to use the formal language with your students, it will be easier for the moment if we use those terms. Now, don't panic — we've just done sets and elements! In table 2, above, each column header is a set, and the headings along the left are elements.

In figure 1, the circles and the words inside them look like sets and elements, respectively — but they're not. In fact, each circle is an element and each thing inside the circle is a characteristic of that element. Each kind of animal is an element; each characteristic represents a set to which that element belongs — for example, the set of all animals that can swim.

A classification diagram turns this inside out.

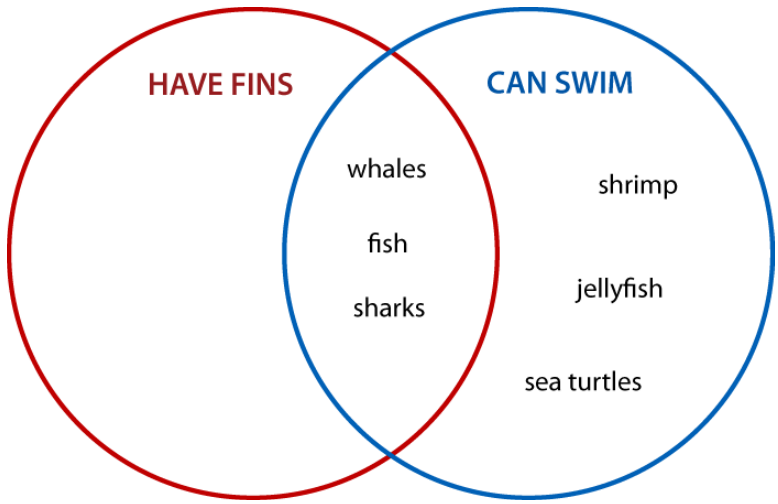

Figure 5. Figure 4. Classifying animals that live in water (two sets).

Now, each circle is a set — the set of all animals that can swim, and the set of all animals that have fins — and the animals themselves are elements of those sets.

On the surface not much has changed, but I want you to stop and think about this a moment because it's actually very powerful! In the first example (figure 1), we were comparing two animals and recognizing how they're alike and different. That's fine, but if we want to compare different animals, we have to start all over. And maybe the words we used to describe those two animals won't work to describe the new animals. Or, to take my serious literary example, when you read two more novels, you'll have to compare the heroes separately. There's no connection to what you've done before. We're not activating students' prior knowledge.

But now, with the classification diagram, we have categories that we can use again and again. So as we study more animals, we can keep adding animals — elements — to these sets. We have a way of understanding zoological *concepts*, not just comparing two individual species! Or — in the literary example — we have a way of understanding the hero in literature, not just Jude Fawley and Diggory Venn. So we're really taking our analysis up a level.

Moreover, while both the comparison diagram and those classification charts we made are closed, either conceptually or visually, this diagram is open. A classification diagram invites us, practically demands us, to consider new ideas.

SUBSETS

In figure 4, we have some blank space on the left — there are not, to my knowledge, any animals that have fins but can't swim. I don't like blank space in diagrams like this. If we draw two circles overlapping, we're implying that all the spaces in our diagram could be filled, and this one, apparently, can't. As a result, our diagram suggests something about animals that isn't true. What might be a better way to draw this diagram?

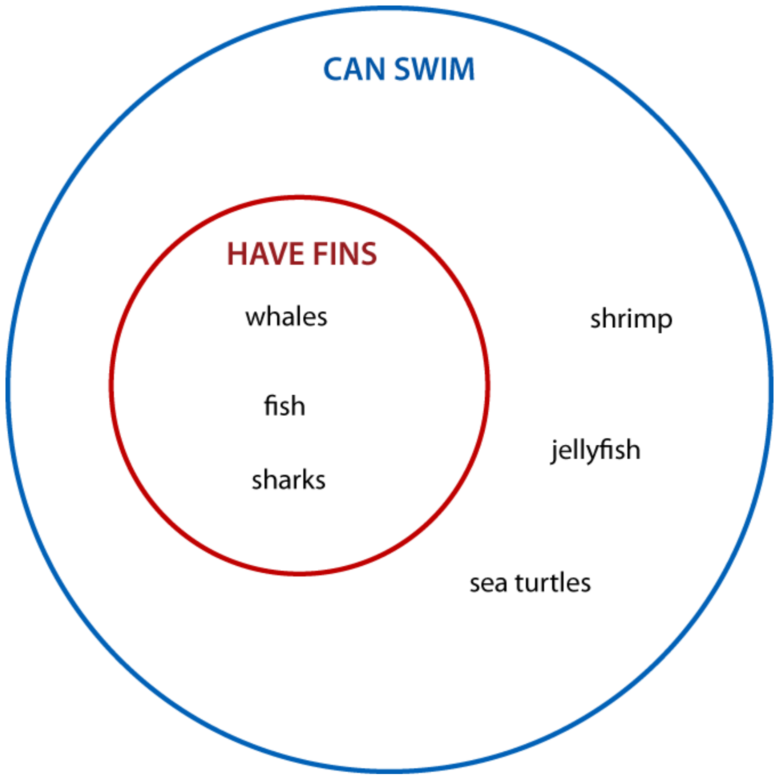

Figure 6. Figure 5. Venn diagram with a subset.

If all animals that have fins can swim, then in mathematical terms, the set of all animals with fins is a subset of the set of all animals that can swim. This diagram, with one circle drawn completely inside the other, makes that relationship clear.

COMPLEX CLASSIFICATION

Once we're freed from stock diagrams and start creating our own, we can do all kinds of complex classification. These diagrams don't just help us keep track of what we already know — they prod us to think and learn more.

Let's add a third category to the diagram. Earlier, we mentioned animals that could breathe in water.

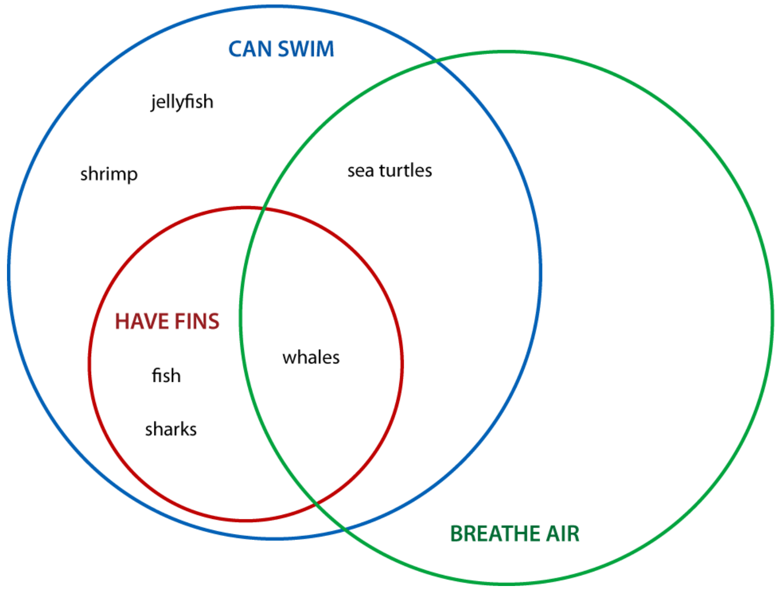

Figure 7. Figure 6. Classifying animals (3 sets).

Again, though, this diagram has blank space. And again, that blank space encourages us to ask whether anything can go there. Did I put the green circle in the right place? Should it overlap each of the others? Can I fill the empty space?

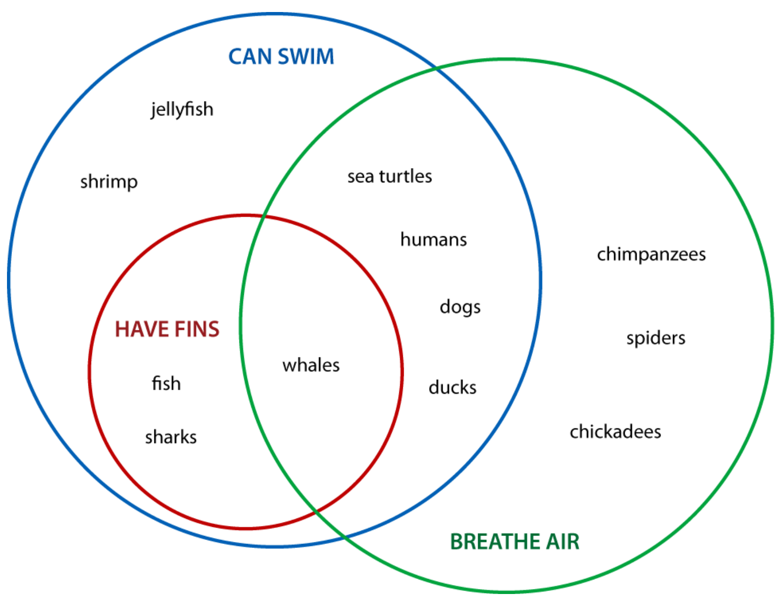

Figure 8. Figure 7. Classifying even more animals (3 sets).

Of course, there are plenty of animals that breathe air and can't swim, such as chimpanzees, spiders, and chickadees. And humans, dogs, and ducks, like sea turtles, breathe air and can swim but don't have fins. We're moving beyond water-dwelling animals now, but that's all right — we're thinking outside the ~~box~~ circle.

Now, this is starting to get interesting. We're inviting all kinds of questions! We're not just describing a particular animal but trying to *think globally* about animals — what kinds of characteristics go together naturally?

Let's look at this fin business, for example. Obviously, some animals that can swim propel themselves with fins, but not all. How do the rest do it? Aha — with legs!

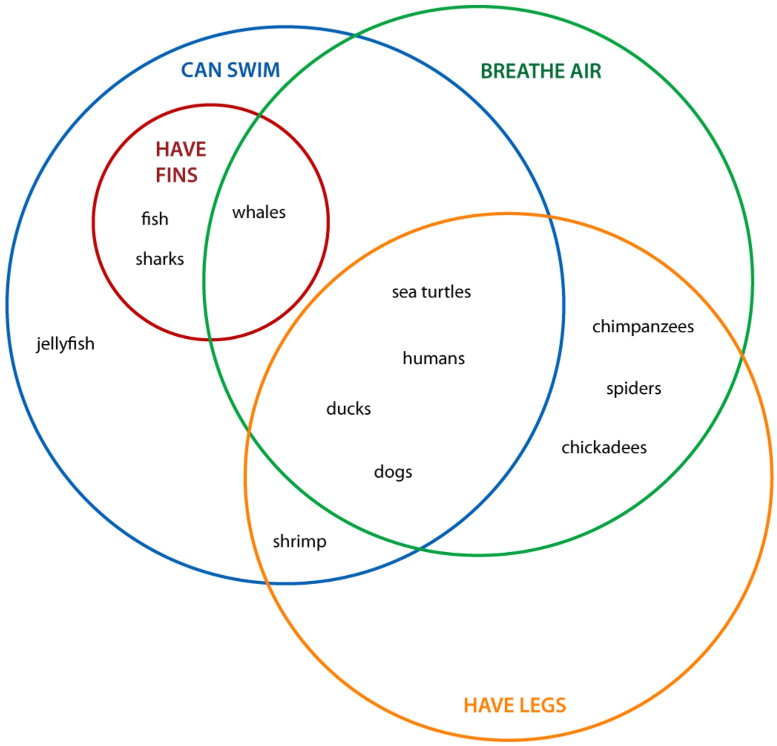

Figure 9. Figure 8. Classifying animals (4 sets).

I've added a circle labeled "have legs" and moved some of our animals into it. Have I put the circle in the right place? That is, can I fill up all the spaces, and do all water-dwelling animals fit on my diagram?

There are three blank spaces in figure 8. Take a minute before you read on and see if you can think of animals that can be classified in these ways:

- animals that breathe air, but cannot swim and do not have legs
- animals that have legs, but do not breathe air or swim
- animals that can swim and breathe air, but have neither legs nor fins

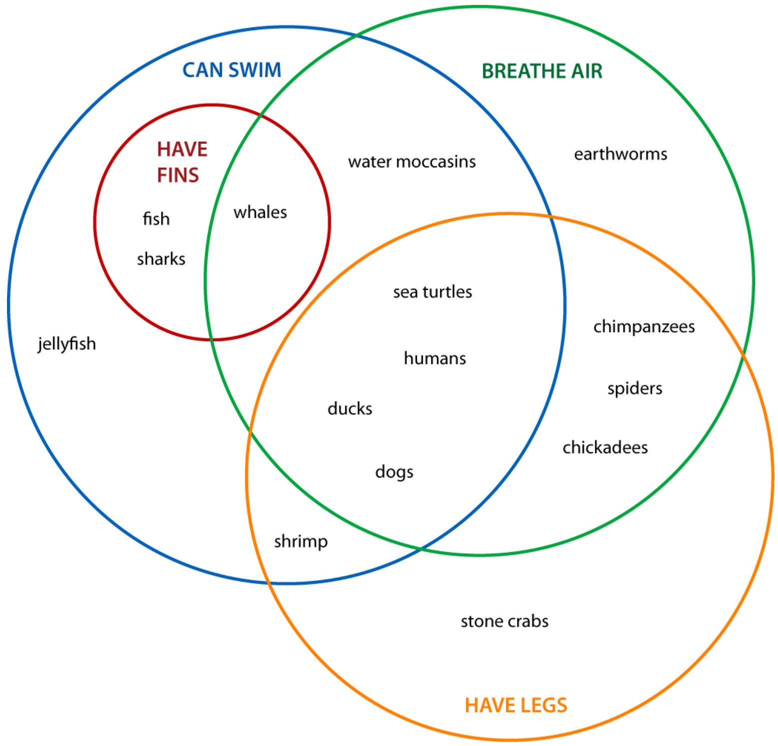

Figure 10. Figure 9. Classifying more animals (4 sets).

Earthworms breathe air, have no legs, and can't swim. Stone crabs have legs, don't swim, and — even though they live in the sand — can't breathe air. And water moccasins (cottonmouths) breathe air and can swim, but have neither legs nor fins.

When you see the finished diagram, it looks easy and obvious, but it took a lot of thinking to come up with these answers. I brainstormed and bugged my colleagues, and still couldn't come up with anything for that last space until a teacher in one of my conference presentations suggested water moccasins. It's harder than it looks, and it's a good collaborative project for students working in groups, because they'll each have different ways of thinking about animals — all of which will be helpful.

THE UNIVERSAL SET

I could keep adding more classifications — "have tentacles," for example. (I'll leave that as a challenge problem for readers: Where would that circle go?) Before we go on, though, let's see whether all the animals we've mentioned so far fit into our diagram. We mentioned sea anemones and oysters — which can't swim, have neither fins or legs, and don't breathe air. Where do *they* go?

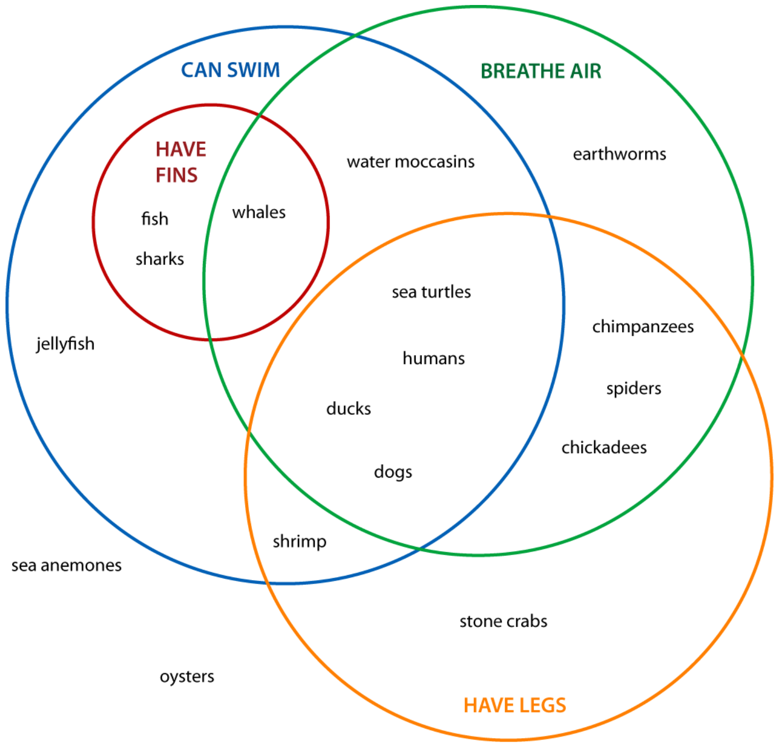

Figure 11. Figure 10. Some animals don't fit.

Oysters and sea anemones "float" in our diagram — they don't seem to fit anywhere. But they're still animals; they're still part of our conversation — they just don't have any of the specific characteristics we've chosen to diagram. We need to include them, because having animals that don't fit helps us to understand our classification system and its limits. Without them, we might think that *all* animals fit into those circles, or that oysters and sea anemones somehow aren't animals.

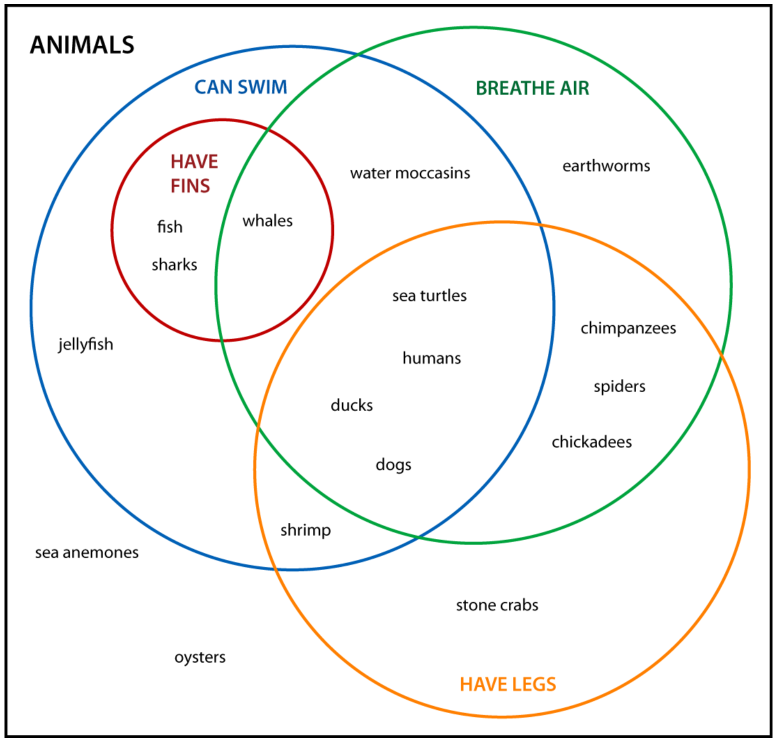

Figure 12. Figure 11. Adding the universal set.

Here, I've simply drawn a box around my diagram and labeled it with the thing we're classifying — animals. In mathematical terms, that box is the *universal set*, the set of all elements in the "universe" of our conversation. The universal set contains everything we might be trying to classify.

Earlier, I said we wanted think outside the box, and we expanded our discussion from sea animals to all animals. But every classroom discussion and every classification system has to have some limits. If we added rocks or clipboards or bagged salad mixes to our diagram, it would just get silly. So we'll define our universal set as the set of all animals, and that's the box we won't think outside. Now, we've clearly defined what we're talking about, and everything we might talk about has a home on our diagram.

The universal set is actually a good place to start: If you're talking about animals, draw that big box at the beginning of the discussion and label it clearly. That will keep the conversation on track and better organize your ideas.

WHEN CLASSIFICATION GETS TOO COMPLEX

We could keep adding classifications to our discussion, but it gets progressively more difficult to add circles to our diagram. Suppose we wanted to add the set of animals that have internal skeletons. Where would *that* circle go? The problem is that until we draw a blank diagram and start trying to fill it up, it's hard to know where the empty spaces are and how we might draw the diagram to eliminate them. But, as I said earlier, we can't draw a diagram with four circles, each of which partially overlaps each of the others. It's mathematically impossible — why that's true is something I'll leave as a challenge problem for advanced readers. (If you don't believe me, get out a pencil and a piece of paper, and try to draw The rest of you will have to trust me: If we want to add a fourth set that isn't a subset of one of the others, we're going to have to move beyond circles.

Edwards-Venn diagrams

The classic Venn diagram, with three circles, was invented by the English philosopher and mathematician John Venn in 1881. His goal was to find symmetrical figures that were elegant and attractive, and he was never satisfied with his attempts to find figures for mapping four, five, and more sets. But a hundred years later, Anthony Edwards, a statistician, geneticist, and evolutionary biologist at Cambridge University, thought of a way to do it.

Figure 13.

Edwards' inspiration was a tennis ball. If you want to divide a tennis ball in half, there are four ways you can do it:

- Divide the top and bottom hemispheres.
- Divide the left and right hemispheres.
- Divide the front and back hemispheres.
- Separate the two pieces of fabric used to create the cover.

If you imagine flattening out a tennis ball and dividing it in each of those four ways, you get a diagram something like this:

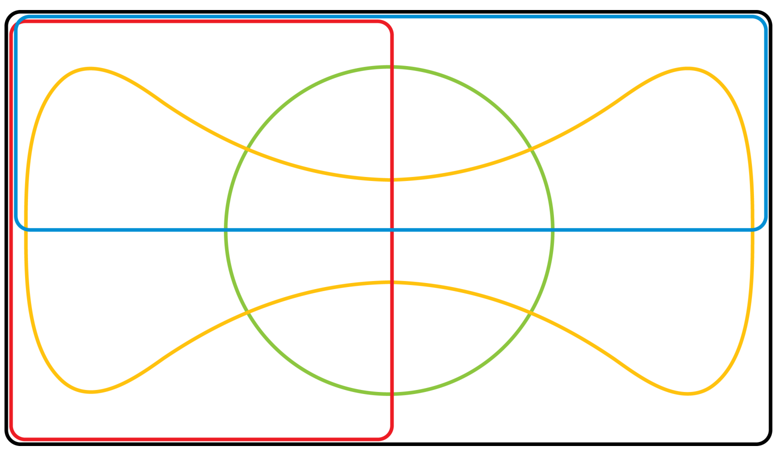

Figure 14. Figure 12. A 4-set Edwards-Venn diagram.

The top (blue) rectangle is the top hemisphere of the tennis ball, the left (red) rectangle is the left hemisphere, the green circle is the front of the ball, and the orange dog-bone shape resembles one of the two pieces of the ball's cover.

Let's use an Edwards-Venn diagram to add "have internal skeletons" to our classification system for animals.

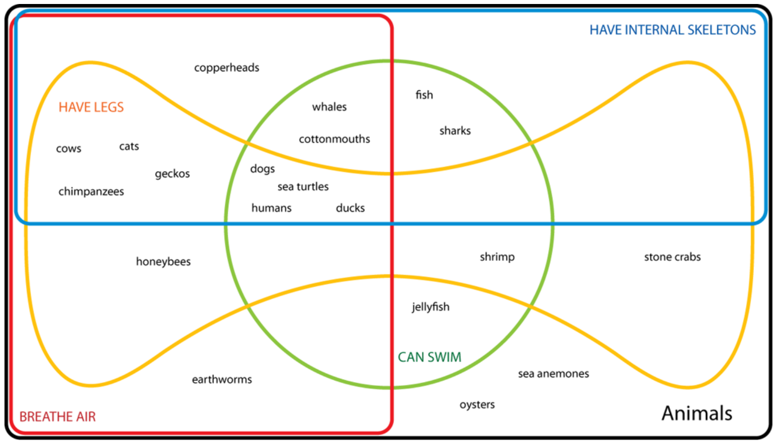

Figure 15.

Figure 13. Classifying animals with a 4-set Edwards-Venn diagram.

There is a lot of blank space on this diagram. Again, the blank space encourages us to think about the relationships among our categories. I couldn't think of any animals that have legs, swim, and breathe air but have no internal skeletons, for example, but I'm not an expert on zoology — and trying to fill in those blanks would make a great challenge problem for advanced students.

Another challenge — harder than it seems — is coming up with four characteristics of animals that are totally independent. If all four characteristics are independent of one another, then we should be able to place animals in each of the spaces in our diagram. Most of the characteristics I thought of seemed to be dependent on others or exclusive of others. We already found, for example, that "have fins" is dependent on, or a subset of, "can swim"; any animal with fins can swim, so those two characteristics aren't independent of each other. "Lay eggs," from our classification chart (table 2), doesn't seem to be fully independent of the others, either, though I'll leave it to you to decide whether that's true. The process of answering those questions, in any case, is as interesting as the answers themselves — they lead to discussion of differences and similarities among mammals and fish and reptiles, body structures, convergent adaptation, and so on.

I did come up with two diagrams that classify things into four independent sets. The first diagram classifies foods based on various standards of healthfulness — low in fat, high in protein, high in fiber, and vegetarian. The second I used to win an argument about whether figure skating is a sport. (Please: No emails. If you don't like it, make your own diagram!)

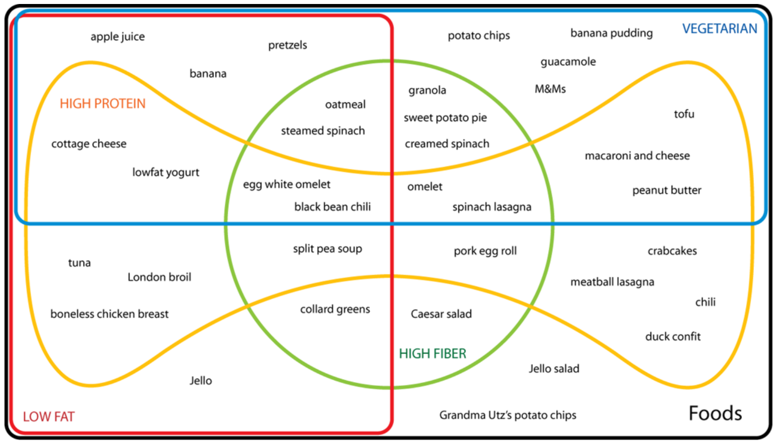

Figure 16.

Figure 14. Classifying potentially healthful foods with a 4-set Edwards-Venn diagram.

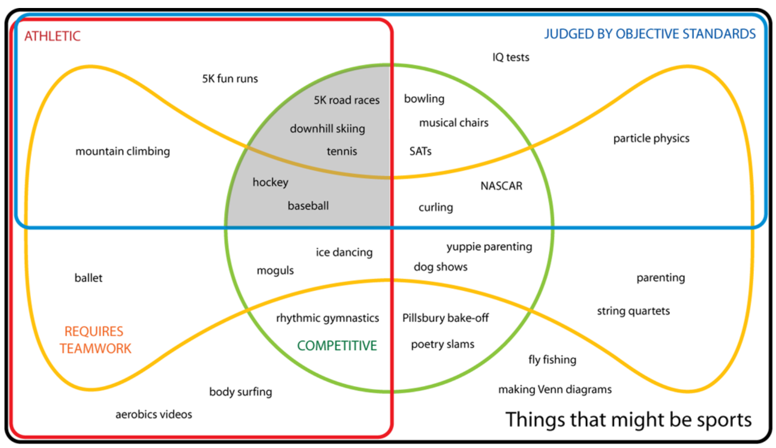

Figure 17.

Figure 15. Classifying activities that might be sports. My definition of a "sport" is the represented by the shaded area.

If you want to try your own, here's a blank diagram[1] (PDF, 8.5"x11") you can download and print.

FIVE SETS, SIX SETS, MORE

Interestingly, if you needed a fifth set, you could draw a sort of gear that looked like the dog bone but had four spokes instead of two — and then for a sixth set you could draw one with eight spokes, and so on.

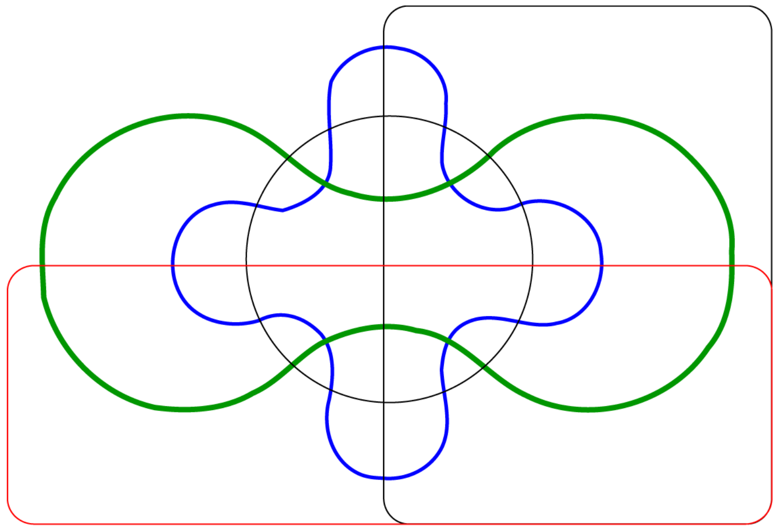

Figure 18. Figure 16. A 5-set Edwards-Venn diagram.

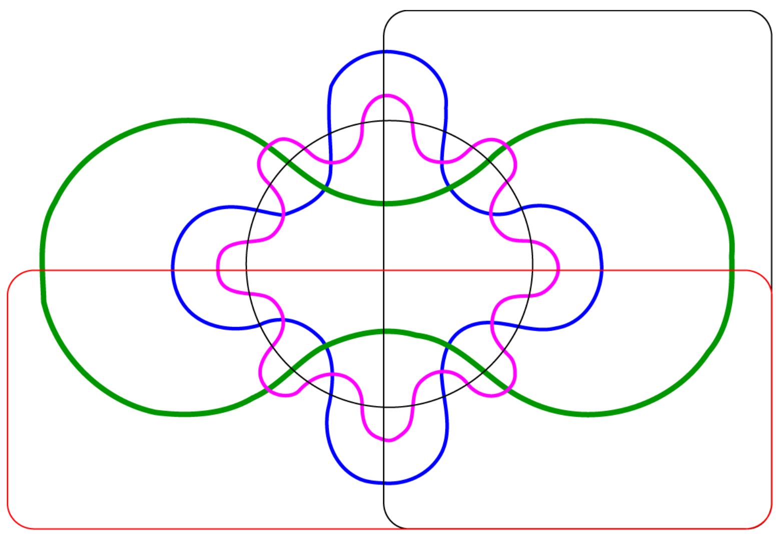

Figure 19. Figure 17. A 6-set Edwards-Venn diagram.

To use this in the classroom, of course, you'd need a really big sheet of paper. But you could reproduce it with felt-tip markers on a roll of butcher paper, hang it on a bulletin board, and use Post-It notes to populate the diagram.

Mad diagramming!

Once you've freed yourself from the double bubble chart, Venn-like diagrams can be used for all kinds of purposes. Here are a few practical diagrams you might use in the classroom.

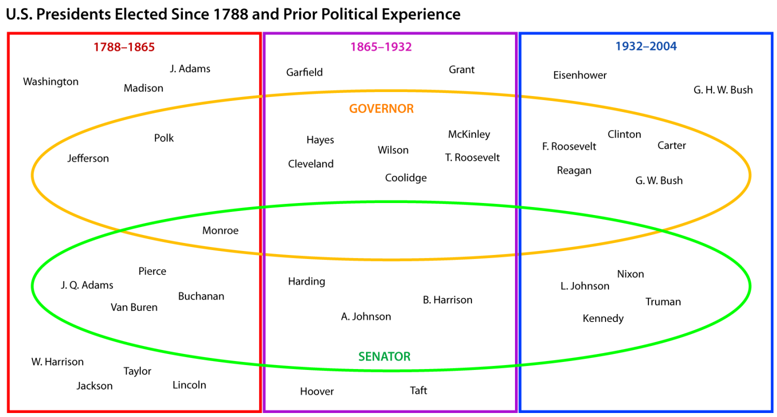

Figure 20.

Figure 18. Classifying U.S. Presidents by era abd prior political experience.

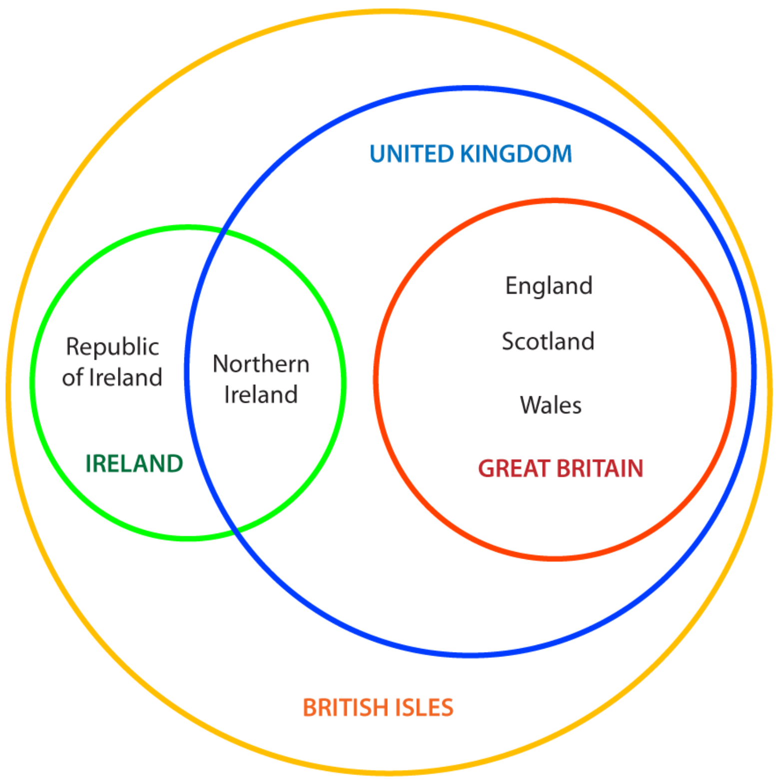

Figure 21. Figure 19. Mapping the islands and countries of the British Isles.

In the classroom

It would be easy to look at the complex diagrams toward the end of this article and say "my kids can't do that." But they can, if you work slowly and use a little creativity.

MAKING THE DIAGRAMS

I created the diagrams for this article in Adobe Illustrator, but you don't need expensive software. You could use Microsoft Word, or any drawing program. You could use a whiteboard with colored markers and Post-It notes. For lower grades, instead of Post-It notes, use index cards with pictures. To get kids out of their seats, use string, colored masking tape, or hula hoops on the floor. Make the circles big enough, and the students themselves can represent the elements!

SCAFFOLDING UP TO HIGHER-ORDER THINKING

Depending on the grade and subject you teach, you'll be able to use complicated graphic organizers like these in different ways and to varying degrees.

At a basic level, you could do the classification work yourself, make a blank diagram, give it to students with a list of elements, and have them figure out where the elements go. As a demonstration, you could use a diagram on a whiteboard, then let students work on their own diagrams in groups.

In some cases, you won't want to take class time to develop the diagram. You could make a diagram and print it off as a study guide — the British Isles diagram (figure 19), for example, would be a good study guide for a geography or history class.

To challenge students, don't label the circles. Give them a blank unlabeled diagram with a list of sets and elements (or a blank drawing on a whiteboard and a pile of Post-It notes or index cards) and let them figure out which set is which and where the labels go. This is an especially good activity if the sets in your diagram aren't independent, like our complex animals diagram in figure 11. Since one set is a subset of another, the circles aren't interchangeable as they are in a simple Venn diagram.

For younger students, you can start them off with double bubble charts for comparison — they're used to that, and it comes more naturally. Then move, just as I did, through charts with Xs up to simple classification diagrams. You could expand this gradually over the course of the year. As a class, pick two characteristics, draw circles, and place elements in them. Then try to add a third characteristic and talk about where the circle ought to go. For example, if you're talking about animals in a given biome, give students the challenge of coming up with their own classification system — what categories are most important to your discussion? Why? Can they come up with three truly independent characteristics of animals in that biome? How about four? Something like this makes a good group activity; let them debate it internally, then present their solutions to the class and then talk about which diagrams are more useful in different ways. If you do that, try to mix up the abstract thinkers with visual thinkers — the kids good at math with the artists — because this kind of activity takes both kinds of thinking.

In higher grades, you can jump in more quickly. If you're discussing characteristics of literary heroes in a high school English course, for example, start by comparing a couple of heroes (as we did in figure 2). Have students identify the most important characteristics, then draw a diagram to represent those characteristics and place all the characters you've studied in the diagram. This would be a good way to figure out what actually makes a "hero" in literature; you might need to revise your diagram two or three times, and even then there might be disagreement. You can walk them through the process of creating the diagram just as I made the animal diagram in this article progressively more complex.

Conclusion: Four rules for using Venn diagrams

To sum up, here are four rules for using Venn (and Venn-like) diagrams to encourage higher-order thinking in your classroom.

1. **Use diagrams for classification, not just comparison.**

    By using circles to represent *sets* and placing the elements within them, you can classify large numbers of things rather than simply comparing two or three.

2. **Draw diagrams to meet your needs.**

    Circles don't have to be the same size, and they don't have to overlap — you don't even have to use circles! By drawing custom diagrams for each topic, you can correctly represent relationships among sets or characteristics.

3. **Draw the universal set.**

    Draw and label the universal set — the set of everything you might be discussing. That keeps the discussion within reasonable bounds, and makes a place for everything in it.

4. **Scaffold students up to using progressively more complicated diagrams.**

    If you work your way up slowly, students will learn to use graphic organizers not simply to keep track of knowledge they've already learned, but to push themselves to think about that knowledge in new ways and to learn more.

CREDITS

I originally developed this article as a presentation for the 2006 conferences of the North Carolina Middle School Association conference and the North Carolina Association for Educational Communications & Technology. Bobby Hobgood helped me develop the content, in particular the ideas for using Venn diagrams in the classroom, and co-presented with me.

On the web

Classroom Instruction that Works: Nine essential strategies

http://www.middleweb.com/MWLresources/marzchat1.html

Researchers Robert Marzano, Debra Pickering, and Jane Pollock at Mid-continent Research for Education and Learning (McREL) have identified nine instructional strategies that are most likely to improve student achievement across all content areas and across all grade levels. First on that list is identifying similarities and differences.

Blank 4-set Edwards-Venn diagram

http://www.learnnc.org/lp/media/authors/walbert/venn/venn-edwards-4.pdf

You can use this PDF file as a template.

What is a Venn Diagram?

http://www.theory.cs.uvic.ca/~cos/venn/VennWhatEJC.html

Mathematical definitions and construction from the Electronic Journal of Combinatorics. This article points out that many of the diagrams I've used are actually Euler diagrams, not true Venn diagrams. I've used the more familiar term here in the interest of simplicity for non-mathematicians.

More from LEARN NC

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Notes

1. See http://www.learnnc.org/lp/media/authors/walbert/venn/venn-edwards-4.pdf.

About the author

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David Walbert is Editorial and Web Director for LEARN NC in the University of North Carolina at Chapel Hill School of Education. He is responsible for all of LEARN NC's educational publications, oversees development of various web applications including LEARN NC's website and content management systems, and is the organization's primary web, information, and visual designer. He has worked with LEARN NC since August 1997.

David holds a Ph.D. in History from the University of North Carolina at Chapel Hill. He is the author of *Garden Spot: Lancaster County, the Old Order Amish, and the Selling of Rural America*, published in 2002 by Oxford University Press. With LEARN NC, he has written numerous articles for K–12 teachers on topics such as historical education, visual literacy, writing instruction, and technology integration.

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